

Experimental test of the no signaling theorem

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In 1981 N. Herbert proposed a gedanken experiment in order to achieve by the "First Laser Amplified Superluminal Hookup" (FLASH) a faster than light communication (FTL) by quantum nonlocality. The present work reports the first experimental realization of that proposal by the optical parametric amplification of a single photon belonging to an entangled EPR pair into an output field involving 5×10^3 photons. A thorough theoretical and experimental analysis explains in general and conclusive terms the precise reasons for the failure of the FLASH program as well as of any similar FTL proposals.

The theory of special relativity lies on a very basic hypothesis: anything carrying information cannot travel faster than the light in vacuum. On the other side, quantum physics possesses marked nonlocal features implied by the Bell's theorem. Nevertheless it has been shown theoretically, on the basis of several properties of the Hilbert space that a "no-signaling theorem" holds: one cannot exploit the quantum entanglement between two space-like separated particles in order to realize any "faster-than-light" (FTL) communication [1, 2, 3]. Indeed the quantum theory of communications teaches us that a single particle cannot carry information about its coding basis at the receiving station. Then, in an attempt to break the "peaceful coexistence" between quantum mechanics and relativity, Nick Herbert proposed in 1981 a feasible experimental scheme, i.e. a FLASH machine, the acronym standing for "First Laser-Amplified Superluminal Hookup" [4]. The proposal, based on the amplification by stimulated emission of a photon in an entangled state, elicited a debate among theorists leading at last to the formulation of the quantum "no-cloning theorem" [5, 6]. However, recent studies have shown that the amplifier noise related to the reduction of "fidelity" $F < 1$ implied by the "no-cloning theorem" cannot be held responsible for the failure of the FLASH program, since the realizable "optimal" fidelity can be large enough to provide a sufficiently large signal-to-noise ratio of the output signals [7, 8, 9]. In spite of that, due to the complex and subtle dynamics underlying any real amplification process, a clear and unambiguous theoretical argument on the precise cause of that failure has never been singled out. After so many years a recourse to the experiment and a careful theoretical analysis were held necessary. This will contribute to our understanding on the properties of the distribution of quantum information within a more advanced theory of the cloning process.

The setup proposed by Herbert is reported in Fig.1. Two space-like distant observers, say Alice and Bob share two polarization entangled photons generated by a common EPR source (Crystal 1). Alice detects by the phototubes D_{φ}^A and $D_{\varphi\perp}^A$ the polarization $\vec{\pi}_A$ of

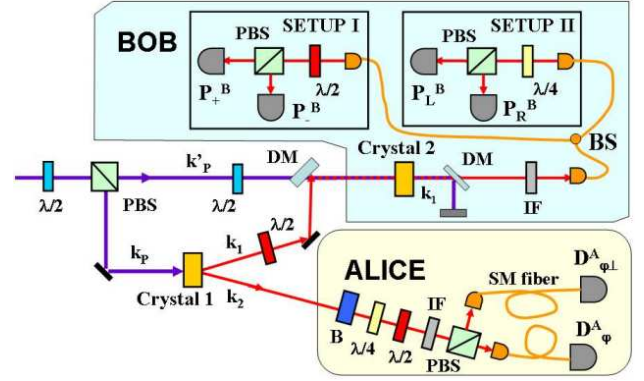


FIG. 1: Configuration of the quantum injected optical parametric amplifier. The polarization entangled photon couple is generated by crystal 1. Crystal 2, realizing the OPA action, is cut for collinear type II phase matching.

her photon in any two orthogonal measurement basis $\{\vec{\pi}, \vec{\pi}^\perp\}_A$. Let us refer for simplicity and with no lack of generalization to the following states: linear *polarizations* $\{\vec{\pi}_\pm = 2^{-1/2}(\vec{\pi}_V \pm \vec{\pi}_H)\}_A$ and *circular* $\vec{\pi}'$'s $\{\vec{\pi}_R = 2^{-1/2}(\vec{\pi}_H + i\vec{\pi}_V), \vec{\pi}_L = 2^{-1/2}(\vec{\pi}_V - i\vec{\pi}_H)\}_A$, where $\vec{\pi}_H$ and $\vec{\pi}_V$ are horizontal and vertical $\vec{\pi}'$'s, respectively. Consider that the choice of the basis is the only coding method accessible to Alice in order to establish a meaningful communication with Bob. If Bob could guess the coding basis chosen by Alice, then a FTL signaling process would be established. Since the detection of an unknown *single* particle cannot carry any information on the coding basis [3], Herbert proposed that Bob could make a "new kind" of measurement on the photon through the *amplification* by a "nonselective laser gain tube". In the jargon of modern quantum optics this consists of a *universal*, i.e. polarization independent amplifier. The amplified beam is then split by a symmetric beam-splitter (BS) so that Bob can perform a measurement on half of the amplified particles in the $\{\vec{\pi}_\pm\}_B$ basis, and on the other half in the $\{\vec{\pi}_R, \vec{\pi}_L\}_B$ basis. The couples of electronic signals registered by two couples of photodetectors are $\{I_+^B, I_-^B\}$ and $\{I_L^B, I_R^B\}$,

correspondingly. In order to challenge the Herbert's proposal, physicists have resorted to the *no-cloning theorem* which forbids the perfect duplication of quantum states. Actually Herbert, aware of the impossibility to produce perfect clones of any input qubit because of the noisy contributions from spontaneous emission, proposed to extract *appropriate signal signature* implying a discrimination between the two measured quantities $\langle |I_+^B - I_-^B| \rangle$ and $\langle |I_R^B - I_L^B| \rangle$ which should depend on the coding basis chosen by Alice.

Let's account now for the experiment. We first selected a definite model for the amplifier by which an experimental test should be carried out. A laser beam at wavelength (wl) $\lambda_P = 397.5nm$ was split in two beams through a $\lambda/2$ waveplate and a polarizing beam splitter (PBS) and excited two non-linear (NL) crystals (β -barium borate) cut for type II phase-matching [7]. Crystal 1, excited by the beam \mathbf{k}_P , is the spontaneous parametric down-conversion source of entangled photon couples of wl $\lambda = 2\lambda_P$, emitted over the two output modes \mathbf{k}_i ($i = 1, 2$) in the *singlet* state $|\Psi^-\rangle_{k_1, k_2} = 2^{-1/2} (|H\rangle_{k_1} |V\rangle_{k_2} - |V\rangle_{k_1} |H\rangle_{k_2})$. The single photon state generated over the mode \mathbf{k}_1 was directed toward Bob's station, simultaneously with a pump beam (mode \mathbf{k}'_P). In virtue of the nonlocal correlation acting on modes \mathbf{k}_1 and \mathbf{k}_2 , the input qubit on mode \mathbf{k}_1 was codified in the polarization state $\vec{\pi}$ by measuring the photon on mode \mathbf{k}_2 in the appropriate polarization basis. The injected photon and the UV pump beam \mathbf{k}'_P were superposed by a dichroic mirror (DM) with high reflectivity at λ and high transmittivity at λ_P .

Let us now describe how the optical amplifier, i.e. the *cloning machine*, has been realized [7]. We adopted a quantum injected - optical parametric amplifier (QI-OPA), exploiting the process of stimulated emission in a NL crystal (crystal 2) in the highly efficient collinear regime, generating a large number of photons over the same direction [10]. This transformation, often referred to as *phase-covariant quantum cloning* (PCQC), realizes cloning with constant amplification for qubits belonging to the equatorial plane orthogonal to the z-axis of the corresponding Bloch sphere [11]. Since they are identified only by a phase φ , the qubits belonging to this plane will be also referred to as: $|\varphi\rangle = 2^{-1/2}(|H\rangle + e^{i\varphi}|V\rangle)$. Note that both the $\{\vec{\pi}_\pm\}$ and $\{\vec{\pi}_R, \vec{\pi}_L\}$ polarization bases do belong to that plane. The adoption of the *optimal* PCQC scheme exhibits the highest quality of the output clones allowed by quantum theory [12].

The interaction Hamiltonian accounting for the parametric amplification of crystal 2, reads $\hat{H} = i\chi\hbar (\hat{a}_H^\dagger \hat{a}_V^\dagger) + h.c.$ and acts on the single spatial mode \mathbf{k}_1 where \hat{a}_π^\dagger is the one photon creation operator associated to mode \mathbf{k}_1 with polarization $\vec{\pi}$ [10]. Its form implies the phase-covariance and the optimality of the cloning process. The multi-photon output state $|\Phi^\varphi\rangle$ generated by

the OPA amplification injected by a single photon qubit $|\varphi\rangle$ is found $|\Phi^\varphi\rangle = \hat{\mathbf{U}}|\varphi\rangle$ where $\hat{\mathbf{U}} = \exp(-i\hat{H}\hbar^{-1}t)$ is the unitary transformation implied by the QIOPA process [10]. Consider any two generic qubits $|\varphi\rangle$ and $|\psi\rangle$ on the Alice's Bloch sphere. They are connected by the general linear transformation: $|\psi\rangle = (\alpha|\varphi\rangle + \beta|\varphi^\perp\rangle)$ with $|\alpha|^2 + |\beta|^2 = 1$. The orthogonal qubits are obtained by the *anti-unitary* time-reversal transformation: $|\varphi^\perp\rangle = \mathbf{T}|\varphi\rangle$ and $|\psi^\perp\rangle = \mathbf{T}|\psi\rangle = (-\beta^*|\varphi\rangle + \alpha^*|\varphi^\perp\rangle)$. In addition, the *unitarity* of $\hat{\mathbf{U}}$ leads to: $\langle\Phi^\varphi|\Phi^\varphi\rangle = 1$, $\langle\Phi^{\varphi^\perp}|\Phi^\varphi\rangle = 0$:

$$\begin{aligned} |\Phi^\psi\rangle &= (\alpha|\Phi^\varphi\rangle + \beta|\Phi^{\varphi^\perp}\rangle); \\ |\Phi^{\psi^\perp}\rangle &= \mathbf{T}|\Phi^\psi\rangle = (-\beta^*|\Phi^\varphi\rangle + \alpha^*|\Phi^{\varphi^\perp}\rangle) \end{aligned} \quad (1)$$

Note that the multi-particle state $|\Phi^\varphi\rangle$, sometimes dubbed "massive qubit", bears exactly the same quantum dynamical properties of the injected single-particle qubit $|\varphi\rangle$ [13]. The average photon number M_\pm^B over \mathbf{k}_1 with polarization $\vec{\pi}_\pm$ is found to depend from the phase φ of the input qubit as follows: $M_\pm^B(\varphi) = \bar{m} + \frac{1}{2}(2\bar{m} + 1)(1 \pm \cos\varphi)$ with $\bar{m} = \sinh^2 g$, g being the NL gain of the OPA. In the analysis basis of circular polarization the expression of the average photon number $M_{R,L}^B$ is the same except for a $\frac{\pi}{2}$ phase φ -shift.

The output state of the crystal 2 with wl λ was spatially separated by the fundamental UV beam through a dichroic mirror, spectrally filtered by an interferential filter (IF) with bandwidth $1.5nm$, coupled to a single mode fiber (SM) and split over two output modes by a beam-splitter (BS). Then each output field was polarization analyzed and detected by a couple of photomultiplier (PM) tubes ($P_{+/R}^B$ and $P_{-/L}^B$). The adopted PM's were Burle C31034-A02 with Ga-As cathode and quantum efficiency $\eta_{QE} = 13\%$. The pulse signals were registered by a digital memory oscilloscope (Tektronix TDS5104B) triggered by $\{D_\varphi^A, D_{\varphi^\perp}^A\}$.

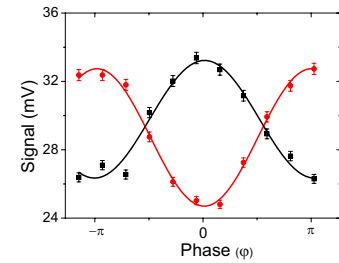


FIG. 2: Average signal versus the phase of the input qubit: I_+^B (square marks), I_-^B (circle marks).

We estimated the experimental gain value $g = (4.45 \pm 0.04)$, which corresponds to a generated mean photon number about equal to 4000 in the stimulated regime. The coherence property of the multiphoton output field $|\Phi^\varphi\rangle$ implied by the quantum superposition character of the input qubit $|\varphi\rangle$ was measured in the

basis $\vec{\pi}_{\pm}$ over the output "cloning" mode \mathbf{k}_1 by signals registered by P_{+}^B and P_{-}^B *conditionally* to single photon detection event by $D_{\varphi\perp}^A$. The corresponding signals I_{+}^B and I_{-}^B , averaged over 2500 trigger pulses, are reported versus the phase φ in Fig.2. The typical fidelity values of the output photon polarization state have been found to be $0.58 \div 0.60$ within the theoretical figures. The discrepancy between experimental and theoretical values can be attributed to the reduced *visibility* of input qubit ($\mathcal{V}_{in} \approx 85\%$) and to the reduced probability $p \simeq 0.4$ of correct amplifier injection under D_{φ}^A -trigger detection. A theoretical model including these imperfections gives rise to an expected value for the visibility $\tilde{\mathcal{V}} = \mathcal{V}_{in} [p(2\overline{m} + 1)]/[p(2\overline{m} + 1) + 2\overline{m}]$ which fits the experimental data obtained for different values of g .

As first step toward the FLASH test, we analyzed the correlations between Alice and Bob's measurements by a *conditional* experiment. Let's make the hypothesis that Bob analyzes his field in the basis $\{\vec{\pi}_{\pm}\}_B$, by the experimental Setup I in Fig.1. By this one the observable $\Delta_{+,-}^B = (I_{+}^B - I_{-}^B)$ was measured over 2500 equally prepared experiments, for different input qubit states $|\varphi\rangle$. As shown by the interference pattern given in Fig.3-a the average difference signal $\langle \Delta_{+,-}^B \rangle$ is equal to zero for an input state belonging to the basis $\{\vec{\pi}_R, \vec{\pi}_L\}$, i.e. for $\varphi = -\frac{\pi}{2}, \frac{\pi}{2}$, while it achieves well defined maximum and minimum values for input qubits equal to $|+\rangle$ ($\varphi = 0$) or $|-\rangle$ ($\varphi = \pi$), respectively. According to Herbert's proposal, the average of the *moduli* of $\Delta_{+,-}^B$, i.e. the value of $\langle |\Delta_{+,-}^B| \rangle$ has been estimated. More precisely, in order to further reduce the effects of the shot-to-shot amplification fluctuations, we registered the average value $\langle |N_{+,-}| \rangle$ with $N_{i,j} = (I_i^B - I_j^B)/(I_i^B + I_j^B)$ for different values of the phase of the input qubit. As shown by Fig.3-b, even in a *conditional* experiment any information on the input state $|\varphi\rangle$ is deleted by the averaging process then making all different input $|\varphi\rangle$ states fully indistinguishable. By a similar procedure, in correspondence with the $\{\vec{\pi}_R, \vec{\pi}_L\}_B$ basis we observed that no information on the basis could be drawn by $\langle |N_{R,L}| \rangle$.

In order to understand the previous results, we investigate the dynamics of the process by considering the probability distribution $\mathcal{P}_{\pm}^{\varphi}(x)$ of the fluctuating observable $x \equiv \delta_{\pm}^B = n_{+}^B - n_{-}^B$ for an input qubit with phase φ . There n_{\pm}^B is the number of photons with polarization $\vec{\pi}_{\pm}$. Assume the *polarization* basis $\{\vec{\pi}_{\pm}\}_A$ corresponding to the *phase* basis $\{\varphi = 0, \varphi^{\perp} = \pi\}_A$. As shown by Fig.4-a, the probability distribution $\mathcal{P}_{\pm}^0(x)$ exhibits a peak in correspondence of $\hat{x}^0 > 0$ while the distribution $\mathcal{P}_{\pm}^{\pi}(x)$ (not reported in the Figure) is identical to $\mathcal{P}_{\pm}^0(x)$ but reversed respect to the axis $x = 0$ with a peak at $\hat{x}^{\pi} = -\hat{x}^0$. The ensemble average values of the observable x are $\langle x \rangle^0 = 2\overline{m} + 1$ and $\langle x \rangle^{\pi} = -(2\overline{m} + 1)$, respectively. Suppose now that Bob keeps his previous measurement basis $\{\vec{\pi}_{\pm}\}_B$ but Alice adopts the new coding basis $\{\vec{\pi}_{L/R}\}_A$

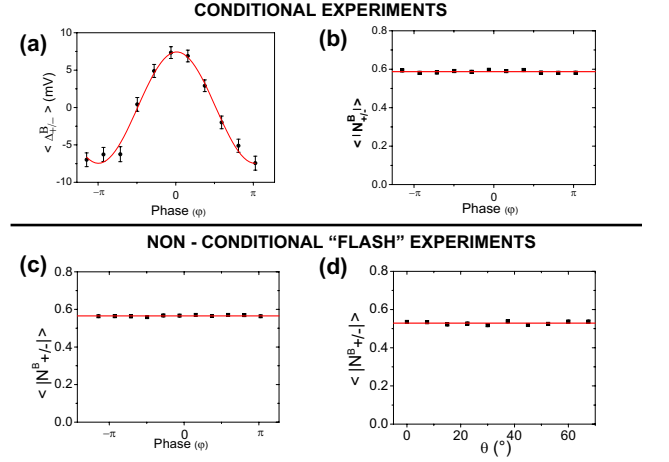


FIG. 3: (a) Mean value of $\Delta_{+,-}^B$ for different input qubit states. (b) Mean value of $|N_{+,-}|$ versus the phase of the input qubit. Error bars are within the corresponding experimental points. (c) Mean value of $|N_{\pm}|$ versus the Alice measurement basis $\{\vec{\pi}_{\varphi}, \vec{\pi}_{\varphi\perp}\}_A$. (d) Mean value of $|N_{\pm}|$ versus the Alice measurement basis: $\{\vec{\pi}_{\theta}, \vec{\pi}_{\theta\perp}\}_A$.

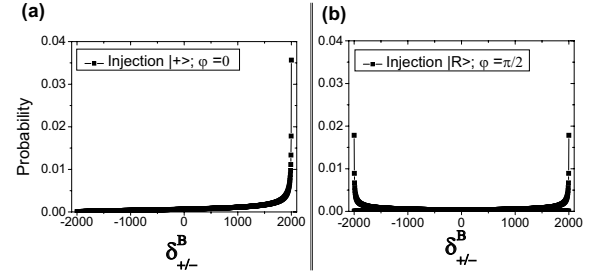


FIG. 4: Theoretical probability distribution of $\delta_{+,-}^B$ for a fixed number of overall output clones $n_{+}^B + n_{-}^B$ equal to 2001. (a) Injected state: $|+\rangle$. (b) Injected state: $|R\rangle$.

$\Rightarrow \{\varphi = \frac{\pi}{2}, \varphi^{\perp} = -\frac{\pi}{2}\}_A$. The average values read now $\langle x \rangle^{\frac{\pi}{2}} = \langle x \rangle^{-\frac{\pi}{2}} = 0$ and the corresponding distributions $\mathcal{P}_{\pm}^{\pm\frac{\pi}{2}}(x)$ are equal for both phases $\pm\frac{\pi}{2}$. Furthermore, Fig. 4-b shows that the two $\mathcal{P}_{\pm}^{\pm\frac{\pi}{2}}(x)$ do not exhibit a single peak centered around the common average value $\hat{x}^{\pm\frac{\pi}{2}} = 0$, as expected for any gaussian-like distribution, but two symmetrical and balanced peaks that are in exact correspondence with the ones shown for \hat{x}^0 and \hat{x}^{π} , i.e. in correspondence with the former Alice polarization basis $\{\vec{\pi}_{\pm}\}_A$. Let's now focus our attention on the probability distribution of $|\delta_{\pm}^B|$ pointing out its implication on FTL communication. From the peculiar behavior of the $\mathcal{P}_{\pm}^{\varphi}(x)$ shown in Fig.4, it is straightforward to conclude that any input qubit leads to the same distribution for $|\delta_{\pm}^B|$. Indeed this unexpected and somewhat counterintuitive result, confirmed by detailed calculations, lies at the basis of the explanation for the FLASH failure. We consider for simplicity and without loss of generality, the average of the square function of δ_{\pm}^B , which is equiva-

lent for our purposes to the average of $|\delta_{\pm}^B|$. It is found $\langle(\delta_{\pm}^B)^2\rangle_{\varphi} = 12\overline{m}^2 + 12\overline{m} + 1$ for any input qubit $|\varphi\rangle$ injected on mode \mathbf{k}_1 pointing out the phase independence of the function. Let us note that the previous results are not invalidate by the low detection efficiency η . Indeed the observable $\langle(\delta_{\pm}^B)^2\rangle_{\varphi}$ behaves as \overline{m}^2 , corresponding to a super-poissonian statistics of δ_{\pm}^B . At variance with the squeezing phenomenology, its fluctuation properties can be investigated with $\eta \ll 1$.

Let's now account for the set of experiments suggested by the FLASH proposal. In this case, since no classical information channel should connect Alice to Bob, a *non-conditional* experimental configuration is necessary. This test was realized by adopting two different triggering solutions for the registering setups at Bob's site: (1) The trigger was provided by the output of a XOR gate fed by the output TTL signals registered by $\{D_{\varphi}^A$ and $D_{\varphi\perp}^A\}$. This device ensured a good discrimination against noise, but any information about Alice's basis was erased. (2) Any channel of information between Alice and Bob was accurately severed. Apart from noise, the two solutions led to identical results. Two experiments were realized. (a) The average value of $|N_{\pm}|$ was measured with Alice measuring her photon polarization in any basis $\{\vec{\pi}_{\varphi}, \vec{\pi}_{\varphi\perp}\}_A$ with $\vec{\pi}_{\varphi} = 2^{-1/2}(\vec{\pi}_H + e^{i\varphi}\vec{\pi}_V)$: Fig. 3-c. (b) The value of $\langle|N_{\pm}|\rangle$ was measured with Alice measuring her photon polarization in any basis $\{\vec{\pi}_{\theta}, \vec{\pi}_{\theta\perp}\}_A$ belonging to the equatorial plane orthogonal to the y-axis. There: $\vec{\pi}_{\theta} = (\cos\theta\vec{\pi}_H + \sin\theta\vec{\pi}_V)$: Fig.3-d. In both cases the observable $|N_{\pm}|$ was found to be independent from Alice's choice of measurement basis.

All these results may be understood in very general and conclusive terms on the basis of the fundamental *linearity* of Quantum Mechanics. Suppose that the Alice's polarization coding bases consists of any generic couple of qubit sets: $\{\vec{\varphi}, \vec{\varphi}_{\perp}\}_A$ and $\{\vec{\chi}, \vec{\chi}_{\perp}\}_A$. In any *non conditional* experiment Bob must carry out his measurements on the state he receives, which here are either one of the two *mixed* mesoscopic states: $\rho_B = \frac{1}{2}(|\Phi^{\varphi}\rangle\langle\Phi^{\varphi}| + |\Phi^{\varphi\perp}\rangle\langle\Phi^{\varphi\perp}|)$ or $\rho_{B'} = \frac{1}{2}(|\Phi^{\chi}\rangle\langle\Phi^{\chi}| + |\Phi^{\chi\perp}\rangle\langle\Phi^{\chi\perp}|)$. However, on the basis of Equation 1 is found: $\rho_B = \rho_{B'} = \hat{1}$, i.e. a fully mixed states. In other words, referring to previous considerations, the linearity of the Hilbert space requires that the *sum of the probability distributions* $\mathcal{P}_{\pm}^{(\varphi, \varphi\perp)}(x) = \mathcal{P}_{\pm}^{\varphi}(x) + \mathcal{P}_{\pm}^{\varphi\perp}(x)$ of the variable x is invariant respect to the corresponding coding basis $\{\varphi, \varphi\perp\}_A$. Since this result is valid for any measurement setup chosen by Bob, the discrimination of the phase φ is impossible, in spite of the different mean values $\langle x \rangle^{\varphi}$ found for different φ 's in the fringing patterns of Fig.3a. This result, shown by the experimental data of Fig. 3c-d, is general and prevents any superluminal communication based on quantum non-locality.

At last, let's consider again the role of the "no cloning theorem", itself a consequence of the linearity of quantum mechanics. Indeed the limitations implied by the cloning dynamics are not restricted to the bounds on the cloning "fidelity", as commonly taken from granted. They also largely affect the *high-order correlations* among the different clones. In facts noisy but separable copies would lead to a perfect state estimation for $g \rightarrow \infty$. However the particles produced by a cloning machine are far from being mutually independent and the corresponding states are the *worst* ones in view of estimating the reduced density matrix, as shown by [14]. The correlations among the different clones, a consequence of the macroscopic coherence of the output field, are clearly exhibited by the peculiar distribution functions of Fig.4. This interpretation has been confirmed in our laboratory by a side quantum state-tomography experiment based on the technique introduced in [15], showing that the density matrix of a two clones system $\rho_C^{I,II}$ is clearly different from the case of separable clones: $\rho_C^I \otimes \rho_C^{II}$.

In summary, we have presented a conclusive theoretical and experimental investigation of the physical processes underlying the failure of the FLASH program. Since the relevant quantum informational aspects of the correlations implied by the cloning process have been little explored in the literature, the present work may elicit an exciting new trend for future research. We thank Gerd Leuchs, Wojciech Zurek, Giancarlo Ghirardi, and Hans Briegel for interesting discussions. We also thank Chiara Vitelli, Sandro Giacomini and Giorgio Milani for technical support. We acknowledge support from the MIUR (PRIN 2005) and from CNISM(Progetto Innesco 2006).

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